

Lesson
16

Understanding Functions

8.F.1

Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.



Understand the Standards

Amanda analyzes the tables below. She notices that there is something special about Table 1. Table 1 has a unique output for every input, while Tables 2 and 3 do not have unique outputs for every input.

input	output
1	12
-1	0
3	24
5	36

input	output
-1	2
0	9
4	5
-1	5

input	output
-2	4
-1	1
-1	-1
-2	-4

Words to Know

function
domain
range
independent variable
dependent variable

Table 2 does not have a unique output for every input, since the input -1 has two outputs. Likewise, Table 3 does not have a unique output for every input, since both -2 and -1 have two outputs. Table 1 represents a function, since every input corresponds to a unique output.

A **function** is a rule that assigns exactly one output to each input. The set of inputs is called the **domain** of the function and is often denoted by the variable x . The set of outputs is called the **range** of the function and is often denoted by the variable y .

Sometimes, the input of a function is referred to as the **independent variable** because it can be any number. The output is referred to as the **dependent variable**, since its value depends on the input. Think of the function $f(x) = 60x$ that shows the distance a car travels in x hours. The input is x hours and x can stand for any value. The output, $f(x)$, is the distance, and it depends on number of hours x .

When talking about a function, it is helpful to name it. You can name a function using a single letter, such as f . The outputs of a function f are written as $f(x)$ which is read as “ f of x .”

Some functions break down for certain inputs. Think about the domain of the functions below. What values of x might cause problems?

$$f(x) = \frac{1}{x} \quad g(x) = \sqrt{x - 10} \quad h(x) = 8x + 2$$

The function $f(x)$ does not work if $x = 0$, because you cannot divide by zero, so its domain is the set of all real numbers except $x = 0$.

The function $g(x)$ does not work if $x < 10$, because you cannot square root a negative value and get a real solution, so its domain is the set of all real numbers ≥ 10 .

The function $h(x)$ works for every value of x , so its domain is the set of all real numbers.



Guided Instruction

The rule “Cube a number and subtract 1 from the result” is a function. Show the function in an equation, table, and a graph.

Step 1 Let $f(x)$ stand for the function. Translate the words “Cube a number and subtract 1 from the result” into words.

$$f(x) = x^3 - 1$$

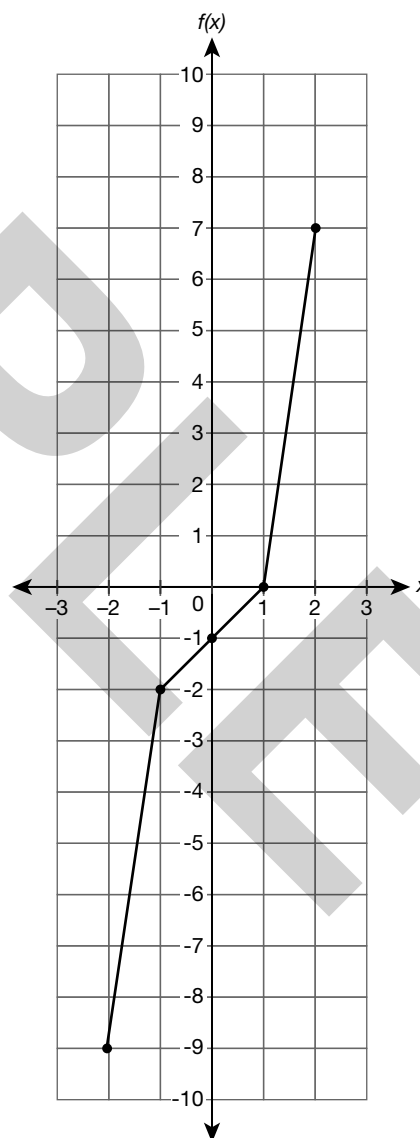
Step 2 Complete an input-output table for the function. Let $x = -2, -1, 0, 1,$ and 2 .

Substitute each value of x into the expression $x^3 - 1$ to solve for $f(x)$.

x	$x^3 - 1$	$f(x)$
-2	$(-2)^3 - 1 = -9$	-9
-1	$(-1)^3 - 1 = -2$	-2
0	$(0)^3 - 1 = -1$	-1
1	$(1)^3 - 1 = 0$	0
2	$(2)^3 - 1 = 7$	7

Step 3 Graph the function. Let the horizontal axis equal x and the vertical axis equal $f(x)$. Graph the points for $(x, f(x))$.

The *graph of a function* is the set of ordered pairs consisting of an input and the corresponding output.





On Your Own

Make an input-output table for each function with integer inputs between 0 and 4.

1. $f(x) = \frac{1}{x+3}$

input	output

2. $g(x) = -4x + 12$

input	output

Use what you now know about functions to decide whether each description represents a function.

3. The input is the day of the year. The output is the average temperature in Chicago on that day.

4. The input is any real number. The output is 15 more than the opposite of the input.

5. The input is any real number. The output is a number that is 6 units away from the input on a number line.

6. The input is the area of a rectangle. The output is the perimeter of that rectangle.

Answer the questions. Share your ideas with a classmate.

7. Does the rule “Add 5 to the input and square the result” define a function? Why?

- Elevate** 8. Which input(s) give the same output for the functions $f(x) = 5x - 2$ and $g(x) = 2x - 5$?

Answer the questions below.

9. Which of the following is not a function?

- A. $f(x) = 3$ B. $f(x) = 5x - 11$
C. $f(x) = x^2$ D. $f(x) = \pm x$

10. Which statement is true about the rule “Subtract 8 from the input and divide by 4”?

- A. The rule is not a function, because the input 0 produces two outputs.
B. The rule is a function, because each input gives exactly one output.
C. The rule is not a function, because some inputs have fractional outputs.
D. The rule is a function, because each input gives an output that is a multiple of four.

11. When $x = -4$, what is the value of $h(x)$ in the equation $h(x) = -4x^2 + 2x - 3$?

12. Does the rule “Find a number more than two times larger than the input” define a function? Why?

- Elevate** 13. The function f is defined as $f(x) = x^2 + 2x + 1$. Find $f(n + 3)$ and $f(n) + f(3)$. Does $f(n + 3)$ equal $f(n) + f(3)$? Explain why or why not.
