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Differentiated Math

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INTRODUCTION

The increased rigor required by the Common Core State Standards for Mathematics is one more argument for fostering a truly differentiated instructional experience. Our students enter the classroom with a variety of interests, strengths, and weakness in mathematics. The Common Core State Standards for Mathematics emphasize focus and depth of understanding. It is this focus that provides an excellent opportunity for teachers to differentiate instruction as each student progresses through the key ideas within a concept. Furthermore, the Standards for Mathematical Practice and their emphasis on application of mathematics to "real-world" situations can help to create an environment in which students engage with mathematics on a more personal level.

As mathematics teachers aim to meet the needs of all their students, they must first acquire the necessary information about their students through thoughtful formative assessments and then determine the best way to create multiple pathways for individuals to meet the same end goal—mastery of the content standards and experience with all the practice standards.

Britt and Tomlinson write in "Common Core State Standards: Where Does Differentiation Fit?" (2012) that the Common Core State Standards "provide *what* students need to be successful... Teachers provide how—the pathway for getting students there." They also note that teachers must use their expertise to select strategies appropriate to *content* and *adapt* strategy to help each student succeed (Britt & Tomlinson, 2012). It goes without saying that this kind of teaching requires creative classroom instruction. However, teachers transitioning to and implementing the new standards should not be discouraged. There are highquality formative assessments and instructional resources readily available to teachers.

In this white paper, we will explore the following questions:

- How do we integrate formative assessment into everyday mathematics instruction in order to support differentiation?
- How do we create a differentiated environment where students are learning mathematics concepts in a way that makes

sense to them and where they are getting the individualized instruction they need most?

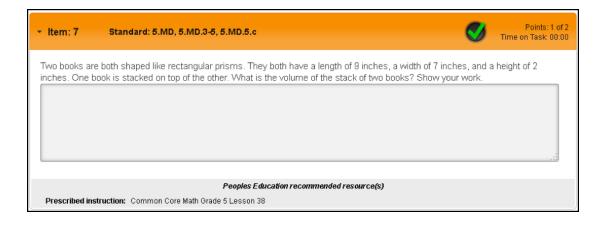
- How does a *blended* or *flipped* classroom support differentiation in mathematics?
- What tools can teachers use to create a flexible curricular path to meet the mathematics standards?
- How do we integrate the Standards for Mathematical Practice into a differentiated classroom?

How do we integrate formative assessment into everyday mathematics instruction to support differentiation?

Formative assessment, the means by which teachers gather data about their students' understandings before instruction begins, is a necessary component of differentiated instruction. Genuine formative assessments allow teachers to support their students wherever they are in their understandings of mathematical concepts and however they best learn concepts. "Formative assessments . . . are essential. They permit the teacher to grasp the students' preconceptions, understand where the students are in the 'developmental corridor' from informal to formal thinking, and design instruction accordingly" (Bransford et al., 2000). Good formative assessment practices can come in many forms.

Math teachers often find it most effective to engage in preassessments of broad concepts in order to determine their students' understanding of the key ideas within that concept. Thus, rather than forging ahead with a new unit of study, teachers can determine what their students already know and to what extent they know it within that unit. From a single word problem that includes essential vocabulary, a teacher can determine student familiarity with upcoming material.

Measuring Up[®] to the Common Core provides teacher-designed formative assessments and Diagnostic Practice Tests aligned to PARCC and Smarter Balanced. Prior to teaching students about volume and rectangular prisms, teachers might use the following word problem from a grade 5 *Measuring Up Insight®* Diagnostic Practice Test for Smarter Balanced in order to evaluate their students' understanding of the concept of *volume*.



Starting with a word problem can help teachers assess how much students know. Teachers can encourage students to use both diagrams and number sentences to show their work. Once the teacher gathers data on student comprehension of the concept, all *Measuring Up* to the Common Core assessments include a Prescriptive Answer Guide (PAG) that identifies the standard tested and the *Measuring Up* to the Common Core lesson (either digital or paper-and-pencil) in which the standard is taught.

Regular formative assessment practices should continue throughout a unit of study so that teachers and students together can assess comprehension and mastery of new material. "As teachers help students track their progress, students can tell exactly where they are. A student who knows he's far from meeting a target will realize that he needs additional practice or more scaffolding. And a student who meets a target guickly can tell that she's ready for an additional challenge" (Dobbertin, 2012). Teachers need a relatively large bank of question prompts in order to gauge where students are having difficulty and when they are ready to move on. "Teachers' regular use of formative assessment improves their students' learning, especially if teachers have additional guidance on using the assessment to design and individualize instruction" (The National Mathematics Advisory Panel, 2008). Measuring Up Insight provides teachers with a large bank of assessment items to guide further instruction.

How do we create a differentiated environment where students are learning math concepts in a way that makes sense to them and where they are getting the individualized instruction they need most?

Once we have the necessary information to determine where each student is in relation to the concept being learned, we must create an environment for differentiated instruction. "The intent of differentiating instruction is to maximize each student's growth and individual success by meeting each student where he or she is and assisting in the learning process" (Hall, 2011). First and foremost, we must establish a partnership with our students in the learning process. We can do this by involving them in formative assessment and giving them a variety of ways in which to learn new material. "Good feedback gives students information they need so they can understand where they are in their learning and what to do next-the cognitive factor" (Brookhart, 2008). It is our responsibility to create independently motivated students with opportunities for meaningful independent practice. There are a variety of ways in which teachers can set up a differentiated classroom experience.

"Differentiating instruction through such approaches as smallgroup learning, individual projects, online instruction, and student choice helps students achieve academic excellence, become valedictorians, and enter universities. Without such differentiation, a uniform curriculum taught in largely the same way to all students in the name of equity can frustrate both high-achieving students and their classmates who need extra help in mastering academic content and skills" (Cuban, 2012).

Whichever method of differentiation we choose (small-group learning, individual projects, digital instruction, etc.), it is advisable that:

"Instruction is concept-focused and principle-driven. The instructional concepts should be broad-based, not focused on minute details or unlimited facts. Teachers must focus on the concepts, principles, and skills that students should learn. The content of instruction should address the same concepts with all students, but the degree of complexity should be adjusted to suit diverse learners" (Hall, 2011).

In other words, students can master concepts at varying levels and depths of understanding depending on their entry point and their individual pace with the material.

Once a teacher determines that a concept is either relatively new or has not been mastered by most students, this teacher might choose to conduct an initial whole class lesson on one topic within a larger concept. For example, when teaching the following *Measurement and Data* standard for grade 3:

3.MD. Measurement and Data: Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

 Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram (CCSSM, 2010). ... the whole class lesson should include both an introduction to the concept and key vocabulary (ex. minute, hour, elapsed time) and a demonstration of the essential skills needed to master the concept. In the *Guided Instruction* section of the following grade 3 *Measuring Up* to the Common Core lesson, teachers can guide students through how to read a clock with both verbal and visual explanations.

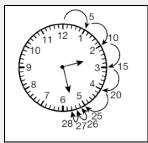
To find the time, first find where the hour hand is. It is between 2 and 3, so the hour is 2.

To find the number of minutes after the hour, count by fives and ones to where the minute hand is pointing.

The time is 28 minutes after the hour.

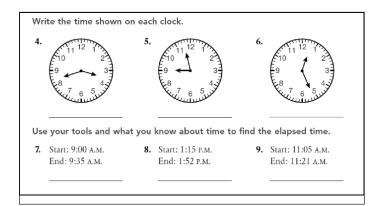
Read the time by saying, "two twenty-eight" or "twentyeight minutes after two."

Write the time as: 2:28.

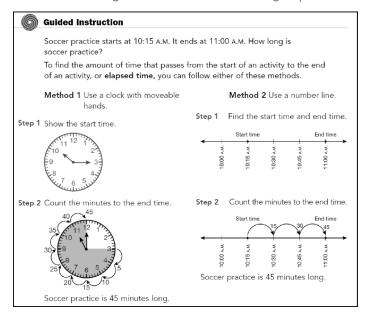


(Measuring Up to the Common Core: Mathematics Level C, 2013, p. 98)

Students are then provided with *On Your Own* problems to work through independently. This is also an opportunity for the teacher to circulate and to assess quickly where students are in their understanding of this concept and what problems they may be encountering.



Students who need further support through small group or one-on-one guided instruction can be teacher-selected or selfselected based on formative assessment feedback. Additional *Guided Instruction* might include more than one method. For example, in the same *Measuring Up* to the Common Core grade 3 lesson, teachers might work one on one or with small groups to review the following two methods for determining elapsed time.



Other students may be ready to embark on meaningful independent study or practice with new skills. Ultimately, all students will be aiming to master the concept and to apply it to challenging questions that situate the concept in the real world. For example, in the following problem, students work in pairs and create their own time problems:

Work with a partner. Pick two activities you do on a school day such as getting dressed or going to school. Think about how much time each activity takes. Write two time problems about your activities where start time, end time, or elapsed time needs to be calculated. Then solve the problems.

(Measuring Up to the Common Core: Mathematics Level C, 2013, p. 101)

And some students may be ready for an additional challenge once they have mastered the appropriate standard. *Measuring Up* to the Common Core provides additional challenges in the *Kick It Up* section at the end of a chapter. Students encounter multistandard, performance-based, collaborative activities that require high-level cognitive thinking. For example, in the following *Kick It Up* challenge, students must apply their mastery of elapsed time to their everyday lives.



Question 1: How can you describe your day using time?

Think of how often you use time each day. You use time to know when to get up, when school starts, when lunch begins, when school ends, and when to go to bed. Plus you use it for many other events during the day.

Work with a partner to describe your school day using time. You should discuss the daily activities and their start and end times. If your schedule varies from day to day, pick one day to use for the project. After you each finish writing the school schedule, work independently to write your own before- and after-school schedules. Keep going until you have included all the major events in your day from waking up to going to bed.

You can continue the activity by making a chart showing the times for all major events in your day for a week. Compare the days to find what events stay the same from day to day and what events change.

(Measuring Up to the Common Core: Mathematics Level C, 2013, p. 101)

How does a blended or flipped classroom support differentiation in mathematics?

For many students the *blended* or *flipped* instructional model can be an effective way of differentiating and individualizing instructional needs. The *blended* instructional model includes a combination of direct instruction and independent digital learning. According to an extensive study of blended learning in educational settings across the country conducted by the Innosight Institute, a non-for-profit think tank, blended learning is defined as a combination of direct instruction and digital learning that must occur in school (brick-and-mortar location away from home) and out of school where students have some control

over time, place, path, and/or pace (Staker, 2011). A *flipped* instructional model occurs when digital delivery of content and instruction is done remotely (often at home after school); students then complete homework in school or at a site where they can get support from a teacher (Horn & Staker, 2012).

In their 2013 EdWeek webinar "Blended Learning Strategies for Common-Core Math," educators Lwanga and Brooks recommend that with either the blended or flipped instructional model teachers use a "dynamic and varied [choice] of technology." This varied technology includes but is not limited to: smart boards, BYOD (bring your own device)/BYOT (bring your own technology), a camera (snapshots of student work for whole class reference), and math games. In the flipped classroom, where homework is completed in school and lessons are completed at home via technology, students can work in a variety of groupings (independently, one on one with teacher, small groups with or without the teacher, or whole class).

Regardless of the model a teacher chooses, e-learning in support of independent learning can occur: in or out of the classroom; self-paced or instructor led; asynchronous or synchronous (in relation to the other students in the class); distance or face to face (with a teacher) (Lwanga & Brooks, 2013). Of course, the amount of blending and the type of student grouping (whole class, individual, one-to-one, small group, independent/selfdirected) depends on what is appropriate to the given grade level and group of students. *Measuring Up* to the Common Core digital allows for 24/7 access (at a teacher's discretion) to diagnostic practice tests, lessons, instruction, and practice so that students can work independently, at their own pace, and on an individualized path of instruction that meets their needs.

What tools can teachers use to create a flexible curricular path to meet the mathematics standards?

Regardless of which model teachers choose in order to create a differentiated classroom, flexibility is critical when grouping students, using technology and resources, and planning for instruction. Flexible instructional planning must ensure that students don't skip critical topics (because it is time to move on) or get stuck in areas of mastery (because the lesson plan indicates a specific topic for that day). One tool that teachers and curriculum planners can use to support this flexibility of instruction is math progressions. "Math progressions help teachers decide where to take instruction, based on what they have observed students being able to do independently and with support" (Trumbull & Lash, 2013).

The University of Arizona's Institute for Mathematics website and Bill McCallum's Tools for the Common Core Standards website both provide mathematical progressions—narrative documents that describe the progression of key ideas across a number of grade levels. Teachers can use these progressions to design a path of study for students to follow independently and to keep students in check to make sure they don't miss a critical key idea. For example in the Overview of K–5, Geometric Measurement Progressions, McCallum explains what students must understand prior to studying volume.

Volume is an amount of three-dimensional space that is contained within a three-dimensional shape. Volume measurement assumes that congruent shapes enclose equal volumes, and that volume is additive, i.e., the volume of the union of two regions that overlap only at their boundaries is the sum of their volumes. Volume is measured by packing (or tiling, or tessellating) a region with a three-dimensional unit (such as a cube) and parts of the unit, without gaps or overlaps. Volume not only introduces a third dimension and thus an even more challenging spatial structuring, but also complexity in the nature of the materials measured. That is, solid units might be "packed," such as cubes in a three-dimensional array or cubic meters of coal, whereas liquids "fill" threedimensional regions, taking the shape of a container, and are often measured in units such as liters or quarts (McCallum, 2012).

The sequence of key ideas within a concept or organizing principle must respect what is known about how students learn (CCSSM, 2010, p. 4). However, the standards do not dictate that key ideas within a concept or organizing principle must be taught in a specific order. Key idea A may be taught after B, or A and B may be taught at the same time depending on the needs of the students and the way in which the teacher feels is it most appropriate (CCSSM, 2010, p. 5). Learning progressions or trajectories can also help teachers to anticipate and to identify common misconceptions students may have and, thus, to shape

feedback—which, in turn, reshapes learning (Sztajn et al., 2012, in Trumbull & Lash, 2013, p. 6). Finally, if a student is having difficulty with a larger concept or even one particular key idea within the concept, teachers can use the progressions to potentially identify the cause of the difficulty.

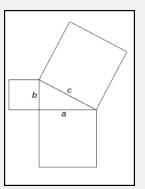
Teachers using *Measuring Up* to the Common Core can move sequentially through the lessons or use the progressions to select lessons where students need additional support. Diagnostic Practice Tests and teacher-created formative assessments (from the *Measuring Up* to the Common Core item bank) also help teachers determine which concepts within a progression on which to focus.

How do we integrate the Standards for Mathematical Practice into a differentiated classroom?

The Standards for Mathematical Practice are an important opportunity to provide students with a considerable amount of personalization in the learning process. Practice standard 1 (making sense of problems and persevering in solving them) and Practice standard 3 (constructing viable arguments and critiquing the reasoning of others) call for students to work individually and in small or large groups and to engage with problems so that they make sense to them. In order to engage with these practice standards, students need opportunities to work at their own pace and in a way in which they can best make sense of mathematics. For some students this process may occur through writing about math; for others it may mean talking their ideas through, or it may mean drawing or creating a model and testing it. Practice standard 1, in particular, relies on a differentiated approach to math. For example, students studying the Pythagorean Theorem can be challenged to make sense of the concept and to explain what they have learned and how it applies to problem solving. In this *Measuring Up* to the Common Core grade 8 geometry challenge, students are asked to make sense of the Pythagorean Theorem through written, visual, and verbal means.

Use what you now know about the Pythagorean Theorem and its converse to answer the questions. Share your answers with a classmate.

- a. If a = 15, b = 20, and c = 25, how will the areas of the two small squares relate to the area of the big square?
 Explain. What does this tell you about the type of triangle formed by the squares?
- b. If a = 9, b = 12, and c = 16, how will the areas of the two small squares relate to the area of the big square?Explain. What does this tell you about the type of triangle formed by the squares?
- c. Construct a diagram similar to the diagram above using side lengths of 3, 4, and 5 inches. Cut out the two smaller squares and cut them into pieces. How should these pieces relate to the big square?



(Measuring Up to the Common Core: Mathematics Level H, 2013, p. 101)



Practice standard 4 (creating models that can function in the real world) helps students to get beyond simply relying on rote procedure and helps them to develop a functioning understanding of math. Students can gain mastery by applying knowledge to real-world situations in myriad ways through openended interaction with mathematical concepts. What emerge are multiple pathways to the same ends. And, teachers can use open-ended tasks to model several ways in which students arrive at a solution to a single problem. Within *Measuring Up* to the Common Core there are *Elevate*, *Critical Thinking*, and *Kick It Up* tasks that support open-ended problem solving. For example, once students have been introduced to the concept of ratios and rates in *Measuring Up* to the Common Core grade 6, they can apply the skill to the following open-ended task.

I2.	Instead of using two or three steps to convert units, it can be done using one equation. Work with a partner to find a method to complete and solve this problem:
Thinking	Convert 2 gallons to ounces.
	$\frac{2 \text{ gallons}}{1} \times \frac{4 \text{ quarts}}{1 \text{ gallon}} \times \underline{\qquad}$
	Make a presentation of your solution method to your class. Will this method work to convert 3 miles to inches? If so, demonstrate how. [Hint: 1 mile = $5,280$ feet.]

(Measuring Up to the Common Core: Mathematics Level F, 2013, p. 32)

Or, students can take on the following Kick It Up task, where they can see the real-world practicality of ratios.

Question 3: How much should we tip?

Wouldn't you like to be able to tell your mom or dad how much they should tip when you eat at a restaurant? The current tipping rates are 15% for acceptable service and 20% for really good service. Let's create a "cheat sheet" for calculating tips.

Create a three-column table. The first column should be a list of prices ranging from \$40 to \$100 in increments of \$5. The middle column will be for a 15% tip and the last column for a 20% tip. Work with a partner and use what you learned about ratios to calculate the tips for each price and record them in your table.

Then come up with a strategy for how you might "guesstimate" the tips for values in between the prices in your table. For example, how much tip would you leave for a \$52 meal based on you calculations for a \$50 meal and a \$55 meal? Share your strategy with your classmates.

(Measuring Up to the Common Core: Mathematics Level F, 2013, p. 32)

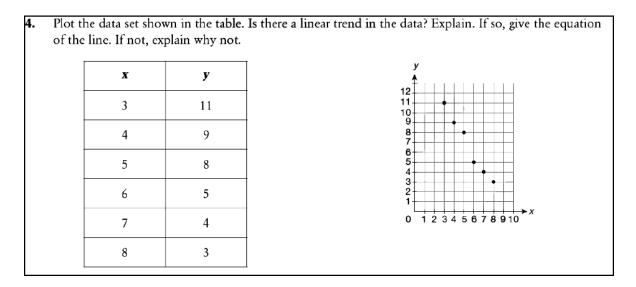
The Standards for Mathematical Practice are not just for students who excel at math or who have mastered a particular topic quickly.

"Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut" (CCSSM, 2010, p. 8).

In other words, engaging all students in open-ended mathematical tasks helps even those who are having difficulty attaining a solid comprehension of a concept.

For struggling math students there are several approaches teachers can use in order to make accessible the more rigorous tasks prompted by the practice standards. One approach is to focus on "abstract reasoning and conceptual understanding using word problems that require less advanced math skills" (Rebora, 2013). In order to do this, students have to understand essential vocabulary and work through problems strategically, and teachers must avoid "algorithms and tricks" (Rebora, 2013). In order to encourage justification of solutions through discussion, students can work in small groups or with their teacher one on one. For example, for those students who might need an additional step before completing the real-world challenge above, teachers might use the following grade 8 *Measuring Up* to the Common Core task to have students explain linear lines in graphing.

Teachers of struggling math students must integrate prior, gradelevel skills into the word problems in order to help students eventually master skills in their grade level. For ELLs, in particular, it is helpful to break word problems apart, annotate sentences, draw model representations, and collaborate on solutions (Rebora, 2013). Many teachers have found that writing and making math applicable in the real world has been a significant help in developing these math reasoning skills for all students.



(Measuring Up to the Common Core: Mathematics Level H, 2013, p. 140---)

CONCLUSION

Whether teachers choose a traditional, blended, or flipped instructional model, *Measuring Up* to the Common Core supports differentiation with individualized pathways and open-ended tasks that allow for personalized responses. Furthermore, the digital version of *Measuring Up* to the Common Core includes a text-to-speech feature in order to provide support for ELLs and special needs students.

Measuring Up Insight provides the teacher-designed formative assessments and an ongoing progress monitoring system through the use of digital or paper-and-pencil Diagnostic Practice Tests aligned to both Smarter Balanced and PARCC assessments. These practice tests are designed based on sample items and test blueprints provided by PARCC and SBAC (grades 1–2 were created from grade 3 test blueprints) and cover all standards, targets, and claims for both PARCC and Smarter Balanced. These practice tests can be used to inform individualized instruction. Each *Measuring Up* to the Common Core Diagnostic Practice Test is accompanied by a Prescriptive Answer Guide (PAG) that identifies the standard tested and the *Measuring Up* to the Common Core lesson in which the standard is taught. All assessments in the program have this feature so that teachers can guide students to focus where they need support.

Measuring Up MyQuest provides a digital, skills-based practice program that is completely student directed, with automatically generated individualized practice. Each three-part lesson includes an introduction to standards and critical vocabulary, guided instruction, and practice. Students can work independently through the content at their own pace, powered by an item bank large enough to deliver all year long. The program automatically scales up or down based on individual performance, meaning that each student is working at his/her optimal level. Furthermore, *Kick It Up* challenges can be used to create individual, small group, or whole group experiences that are cognitively challenging and engage students in the Standards for Mathematical Practice.

Teachers who use the *Measuring Up* to the Common Core program can be assured of a seamless transition to these standards and can benefit from the support of its integrated, essential tools for differentiation in the mathematics classroom.

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