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Student Walk-Through

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Lesson 1 Understand Ratios

Introduction

What Will I Learn?

- How do you describe unit rates?
- How do you create tables of equivalent ratios and plot pairs on the coordinate plane?

You can use ratios to describe a school election. If you say that a person won by a vote of 37 to 26, you're using a ratio!



Break Down the Skills

A **rate** compares two quantities with different units of measurement.

- You can write a rate in words or as a fraction.
- Here are some examples of rates.

\$10 for 5 notebooks
or
 $\frac{\$10}{5 \text{ notebooks}}$



3 cups of oats for 4 servings
or
 $\frac{3 \text{ cups of oats}}{4 \text{ servings}}$



60 miles in 2 hours
or
 $\frac{60 \text{ miles}}{2 \text{ hours}}$



A **unit rate** is a rate in which the second measurement is 1 unit.

- You can write a rate as a unit rate by dividing.

$$\frac{60 \text{ miles}}{2 \text{ hours}} \div \frac{2}{2} = \frac{30 \text{ miles}}{1 \text{ hour}} \leftarrow 30 \text{ miles in 1 hour is a unit rate}$$

A rate is a special kind of **ratio**, which is a comparison of two quantities.

There are two types of ratios. Use this model to compare them.



- A **part-to-part ratio** compares one part of a whole to another part of the same whole.



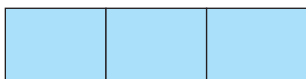
3 **shaded** parts



2 **unshaded** parts

You can write the ratio of **shaded** parts to **unshaded** parts as 3 to 2, 3 : 2, or $\frac{3}{2}$.

- A **part-to-whole ratio** compares one part of a whole to the total number of parts in the whole.



3 **shaded** parts

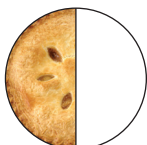


5 **total** parts

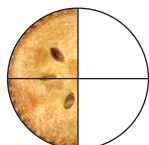
You can write the ratio of **shaded** parts to **total** parts as 3 to 5, 3 : 5, or $\frac{3}{5}$.

Equivalent ratios are ratios that can be simplified to the same value, such as in the same amount of these whole pies.

- The ratios 1 : 2, 2 : 4, and 4 : 8 are equivalent.



1 : 2



2 : 4



4 : 8

You can use the word "per" to describe a rate as in "miles per hour."



You can write the ratio of parts in any order. For example, you could also say that the ratio of unshaded parts to shaded parts is 2 : 3.



You can organize equivalent ratios in a table.

- Then you can write them as **ordered pairs**, which are sets of numbers written in a particular order.
- Remember that the first value in an ordered pair is the **x-coordinate**. The second value is the **y-coordinate**.

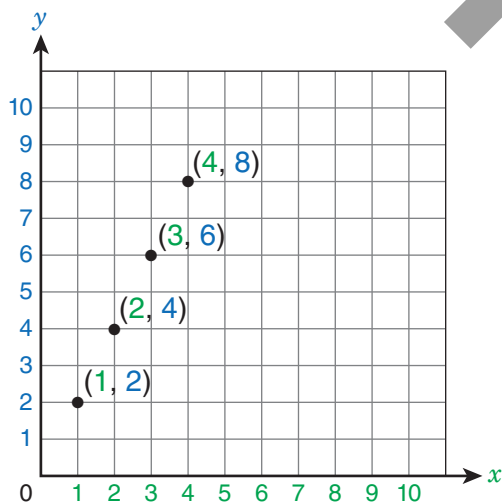
Ratio	(x, y)		Ordered Pair
	x-coordinate First Value	y-coordinate Second Value	
1 : 2 →	1	2	→ (1, 2)
2 : 4 →	2	4	→ (2, 4)
3 : 6 →	3	6	→ (3, 6)
4 : 8 →	4	8	→ (4, 8)

Look for patterns to fill in missing values or add to the table.



You can plot the ordered pairs on the coordinate plane.

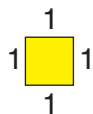
- The x-coordinate tells how far the point is to the left or right of the **origin** (0, 0).
- The y-coordinate tells how far the point is above or below the origin.



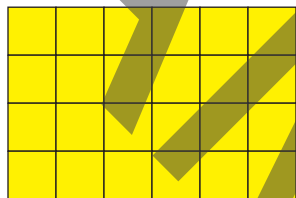
Guided Instruction

Area is a measure of the amount of space taken up by a plane figure.

One way to measure area is by using unit squares. A unit square is a square that has a side length of 1 unit.



- One way to find the area of this rectangle is by counting the number of unit squares inside it.
- Another way to find area is to count the number of unit squares in each row and column. Then multiply the number of rows by the number of columns.



There are each row has _____ unit squares and each column has _____ unit squares.

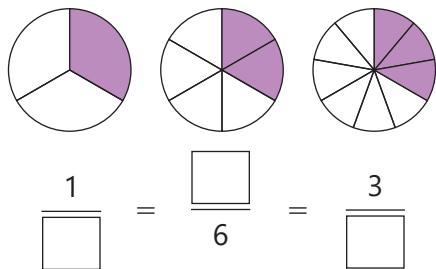
The area of the rectangle is _____ \times _____ = _____ square units, which is the same answer you get by counting.

Count the unit squares. _____ unit squares.

Some areas are fractions instead of whole numbers.

When working with fractions, it often helps to write equivalent fractions. Equivalent fractions are fractions that have the same value even though they have different numbers in them.

Use the model to fill in the equivalent fractions.

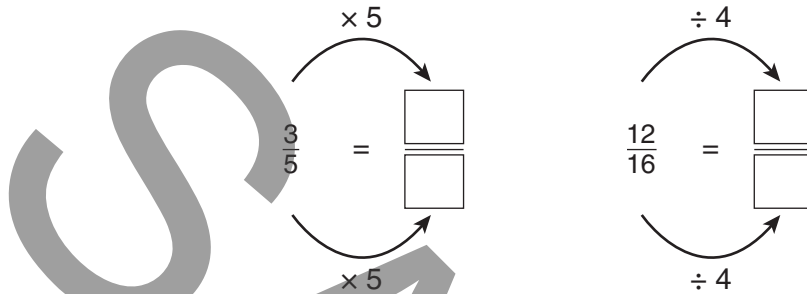


The unit of a unit square can be any unit of length, such as inches or centimeters.



When you do not have images to look at, you can multiply or divide the numerator and denominator by the same number to find an equivalent fraction.

Fill in the missing numbers to find each equivalent fraction.



To write equivalent fractions, do to the denominator what you do to the numerator.



Equivalent fractions: $\frac{3}{5}$ and $\frac{\square}{\square}$ $\frac{12}{16}$ and $\frac{\square}{\square}$

You can use what you know about fractions to describe rates. A rate compares two quantities that are measured in different units.

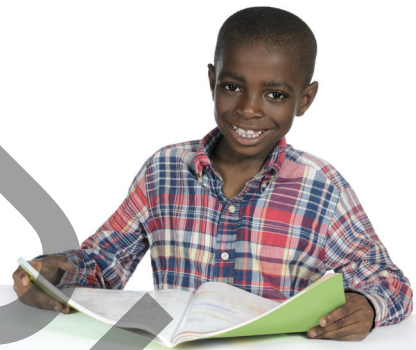
Write a rate for each example.

Elijah read 40 words in 2 minutes.
 words
 minutes

Gasoline costs \$16 for 5 gallons.
 dollars
 gallons

A unit rate is a rate in which the denominator is 1 unit.

To calculate a unit rate, divide one quantity by the other.



The Li family drove 124 miles in 4 hours. Fill in the missing values to find the unit rate.

$$\frac{124 \text{ miles}}{4 \text{ hours}} \div \frac{\square}{4} = \frac{\square \text{ miles}}{1 \text{ hour}}$$

Complete the table to fill in each unit rate.

Rate	Unit Rate
$\frac{48 \text{ beads}}{6 \text{ bracelets}}$	<input type="text"/> beads <input type="text"/> bracelet
$\frac{60 \text{ kilometers}}{5 \text{ hours}}$	<input type="text"/> kilometers <input type="text"/> hour

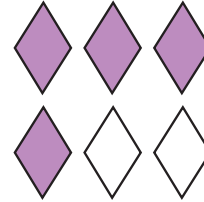
A rate is a kind of ratio. A ratio compares two quantities.

- A rate compares quantities with different units.
- A ratio compares quantities with the same or different units.

Use the model to describe the ratios.

Write **part-to-part ratios**.

- Shaded parts to unshaded parts: _____ to _____
- Unshaded parts to shaded parts: _____ : _____



Write **part-to-whole** and **whole-to-part ratios**.

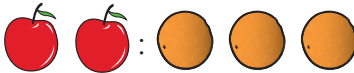
- Shaded parts to all parts: $\frac{\square}{\square}$
- Unshaded parts to all parts: _____ to _____
- All parts to shaded parts: _____ : _____
- All parts to unshaded parts: $\frac{\square}{\square}$

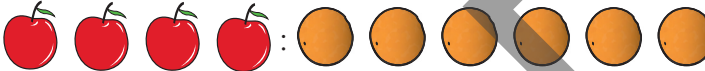
You can write ratios using the word "to," using a colon (:), or using a fraction bar. For example, 1 to 2, 1 : 2 and, $\frac{1}{2}$ are all ratios.



Just as you did with equivalent fractions, you can describe equivalent ratios.

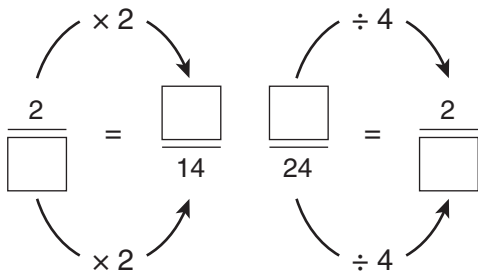
Fill in the equivalent ratios of apples to oranges.

2 : _____ 

_____ : _____ 

You can use multiplication or division to write equivalent ratios.

Fill in the missing numbers to create equivalent ratios.



You can use tables to organize and find equivalent ratios.

The ratio of boys to girls in a class is **3 : 5**. Complete the ratio table to compare boys to girls.

	Boys	Girls
	3	5
$3 \times 2 \rightarrow$		
$3 \times 3 \rightarrow$	9	
$3 \times 4 \rightarrow$		

← 5×2
← 5×3
← 5×4

You can use the values in a ratio table to write ordered pairs.

Complete the equivalent ratio table for the ratio 3 to 2. Then write the ordered pairs.

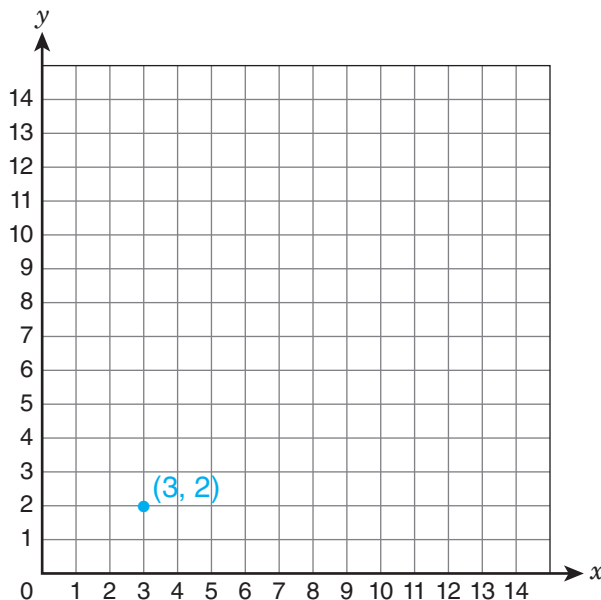
First Value	Second Value	Ordered Pair
3	2	(3, 2)
		(____, ____)
		(____, ____)
		(____, ____)

Remember that in an ordered pair, the first value is the x-coordinate, and the second value is the y-coordinate.



Now that you have ordered pairs, you can plot them on a coordinate plane.

Plot the ordered pairs from the equivalent ratio table on the coordinate plane.



Independent Practice

Answer the questions that follow.

Practice 1

- 1 What is the ratio of shaded parts to unshaded parts?



- A 7 to 12
 B 5 to 7
 C 7 to 5
 D 12 to 7

- 2 Each place setting contains 2 forks and 1 spoon. Circle the ratios that are equivalent to the ratio of forks to spoons. Select the three correct answers.

$\frac{4}{2}$ $\frac{3}{2}$ $\frac{6}{3}$ $\frac{4}{4}$ $\frac{4}{5}$ $\frac{8}{4}$

Start by finding the unit rate of forks to spoons.



- 3 Complete the table to show ratios that are equivalent to $\frac{3}{4}$.

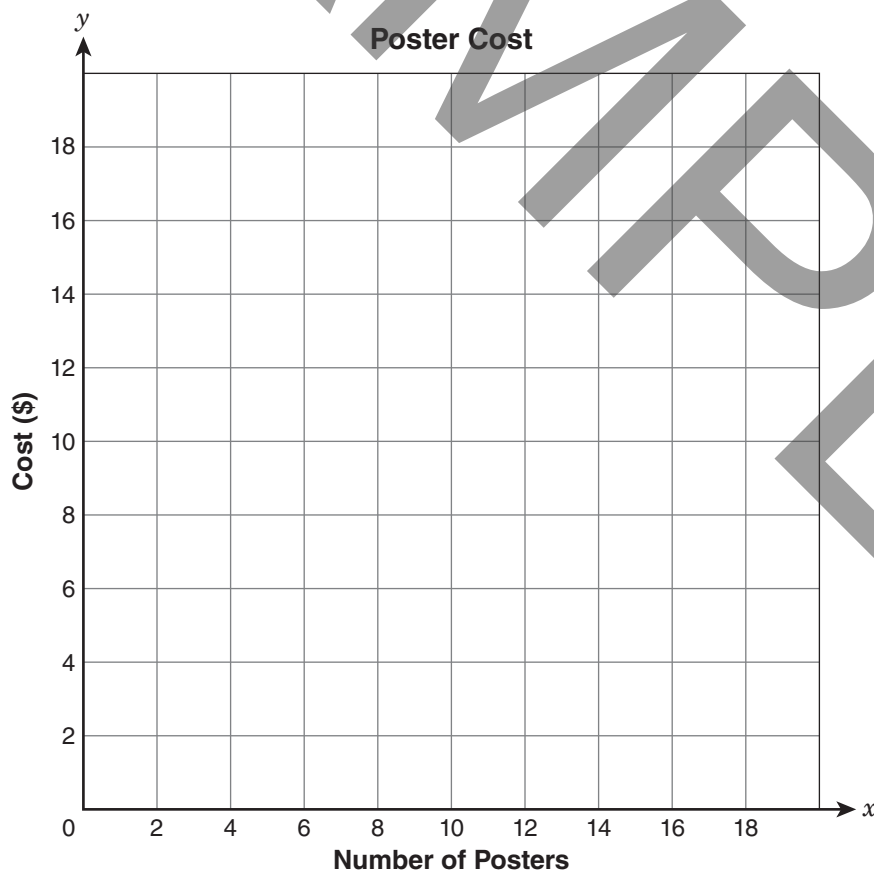
First Value	Second Value
3	4
	20

- 4 Cami earned \$125 for mowing 5 lawns. What is the unit rate for the pay that Cami received? _____

- 5 Use the table to create ordered pairs and plot them on the coordinate plane.

Number of Posters	Cost (\$)
1	4
2	8
3	12
4	16

You can multiply the number of posters by the cost in the first row to find the equivalent ratios.



Practice 2

- 1 What is the ratio of all parts to unshaded parts? Express your answer in simplest form.



- 2 Complete the table to find equivalent ratios for the ratio 5 to 6.

First Value	Second Value
5	6
10	
	24
30	

Use the same steps that you would use to find equivalent fractions to find equivalent ratios.



- 3 Emmanuelle skips rope 105 times in 3 minutes. Rochelle skips rope 120 times in 4 minutes. What unit rate describes each friend's skipping?

- 4 Which ratios are equivalent to $\frac{15}{12}$? Select the three correct answers.

A $\frac{4}{5}$

B $\frac{20}{15}$

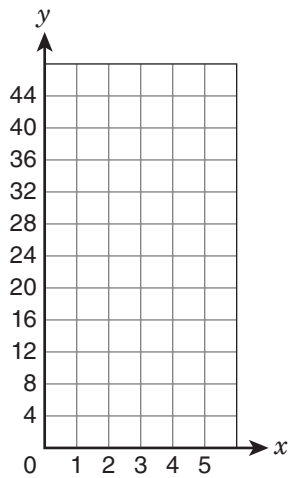
C $\frac{10}{8}$

D $\frac{5}{4}$

E $\frac{25}{20}$

F $\frac{21}{18}$

- 5 Plot the ordered pairs that represent a ratio of $\frac{1}{8}$ for $x = 1, 2, 3, 4,$ and 5.



Exit Ticket

A snowstorm lasted for 5 hours. When the storm ended, 10 inches of snow had fallen. The same amount of snow fell each hour.

What is the unit rate of snowfall in inches per hour?

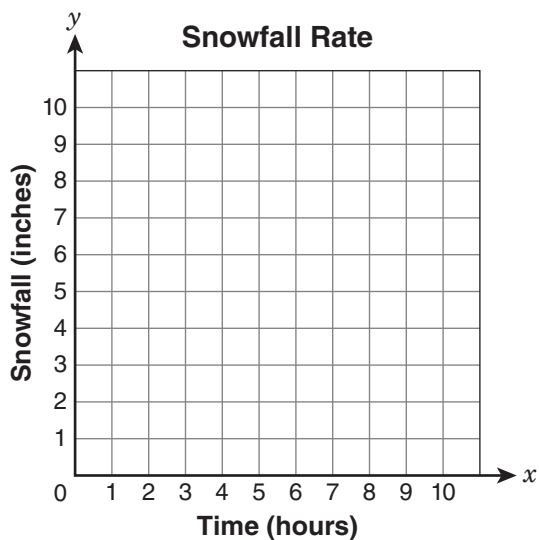
_____ inches per hour

Complete the ratio table to show the number of inches of snow during each of the first 4 hours.

Time (hours)	Snowfall (inches)
1	
2	
3	
4	
5	10



Create ordered pairs from the ratio table and plot them on the coordinate plane.



TEACHER GUIDE

Lesson 1 Understand Ratios

At-a-Glance

Learning Objectives	Review Skills
<ul style="list-style-type: none">• Describe unit rates.• Create tables of equivalent ratios and plot pairs on the coordinate plane.	<ul style="list-style-type: none">• Find area using unit squares.• Find equivalent fractions.
Academic Vocabulary	Why Students May Struggle
rate unit rate ratio equivalent ratio ordered pair x-coordinate y-coordinate origin	Students may list ratios that are not equivalent before finding ordered pairs. Students may add or subtract instead of multiplying or dividing to find equivalent ratios.

WHAT WILL I LEARN?

ACTIVATING PRIOR KNOWLEDGE

- Draw and shade a two-dimensional shape on the board, such as a square or rectangle. Remind students that the size of the shaded space is area.
- Invite students to brainstorm examples of real-life situations involving area, such as the area of a wall for painting and the area of a floor for carpeting.
- Remind students that one way they learned to measure area is by using unit squares. Distribute prepared unit squares to pairs or small groups of students. Have them use the unit squares to measure an area, such as the area of a notebook cover or desktop. Allow students to share their results, and encourage them to use correct units depending on the size of each unit square. Invite volunteers to explain how they found area, and lead students to recognize that they can use multiplication. As needed, model how to count unit squares for length and width and use them to find the product.
- Remind students that they have worked with fractions.
 - Draw a fraction model on the board, such as a rectangle divided into 3 parts. Shade 1 part and write $\frac{1}{3}$ next to the model. Explain that the fraction shows 1 shaded part out of 3 total parts.
 - Beneath the rectangle, draw another rectangle with the same size but this time divide it into 6 parts. Shade 2 parts in such a way that the shaded part aligns with the shaded part of the rectangle above it. Invite a volunteer to write a fraction to describe this model. Lead the student to write $\frac{2}{6}$.

Lesson 1 Understand Ratios

Introduction

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You can use ratios to describe a school election. If you say that a person won by a vote of 37 to 26, you're using a ratio!

Break Down the Skills

A **rate** compares two quantities with different units of measurement.

- You can write a rate in words or as a fraction.
- Here are some examples of rates.

$\$10$ for 5 notebooks or $\$10$ for 3 notebooks

3 cups of oats for 4 servings or 3 cups of oats for 8 servings

60 miles in 2 hours or 60 miles in 2 hours

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- Ask students if both fractions describe the same amount, and confirm that they do. Then ask students for words that describe the same amount and guide them to say the word *equivalent*.
- Challenge pairs of students to draw additional models of equivalent amounts using fraction models. Invite volunteers to share their results.
- To help students review multiplication and division for finding equivalent fractions and ratios, pass out individual whiteboards or have students use a blank sheet of paper.
 - Write the problem $405 \div 3$ on the board. Ask students what methods they can use to find the quotient (standard algorithm, arrays, area models, and so on.) Working together, solve the problem using two different methods.
 - Have students practice dividing whole numbers by 1- and 2-digit divisors. Avoid remainders in the quotients.
 - Remind students that a fraction is an alternative way to write a division problem. Write $\frac{405}{3}$ on the board. Have a volunteer label the numerator and the denominator. Discuss that $\frac{405}{3} = 405 \div 3$.
 - Write the problem 15×4 on the board. Ask students what methods they can use to find the product (standard algorithm, arrays, area models, and so on.) With their help, solve the problem using two different methods.
 - Have students practice multiplying 2-digit whole numbers by 1-digit whole numbers.

EXPLICIT INSTRUCTION

- Write 3 sandwiches for \$15 on the board. Ask students how they can find the cost of 1 sandwich, if the cost of each sandwich is always the same.
 - Explain that to find the cost of 1 sandwich, they can divide. Clarify that one way to divide is to write a fraction that includes the units.
 - Write $\frac{\$15}{3 \text{ sandwiches}}$ on the board. Explain that to find the cost of 1 sandwich, they can rewrite the fraction in simplest form. Write $\frac{\$15 \div 3}{3 \text{ sandwiches} \div 3}$ on the board.
 - Ask students to state the cost of 1 sandwich.
- Remind students that they know that 1 sandwich costs \$5 and 3 sandwiches cost \$15. Explain that they can use the cost of \$5 for each sandwich to find the cost of any number of sandwiches.
- Make sure students understand that what is done to the denominator must also be done to the numerator. Ask students how they can find the cost of any number of sandwiches. Then ask how they can find the number of sandwiches that can be bought for any amount of money.

BREAK DOWN THE SKILLS

TEACH ACADEMIC VOCABULARY

- Read the information about **rates** together. Explain that a rate compares two quantities with different measurement units.
 - Point out that a rate can be written using words or as a fraction.
 - Discuss that measurement units in rates can be units of measure, such as feet or hours, or they can also be a number of items, such as 5 lemons.
 - Invite volunteers to read aloud each of the examples shown.
- Together, read the definition of **unit rate**. Ensure students understand that in a unit rate, the second measurement is 1 unit.
 - Explain that a rate can be written as a unit rate by using division or multiplication. Walk through the example provided by explaining that dividing each measurement by 2 results in a unit rate. Mention that the numeral 1 does not have to be written before the unit of measurement, but it is shown here for clarity.
 - Work with students to determine the unit rate for each of the rates presented above. [$\$2$ per notebook, 0.75 cup of oats per serving, 30 miles per hour]
- Together, read the definition of **ratio**. Note that *ratio* is a more general term than *rate*. Explain that there are two types of ratios.
 - As a group, read through the description of part-to-part ratios. Use the model to show that one ratio describes the 3 shaded parts to the 2 unshaded parts.
 - Have students highlight the three ways to write this ratio.
 - Direct students to read the hint, and then challenge them to write the ratio of unshaded parts to shaded parts in three different ways.
 - Point out that corresponding part-to-part ratios are reciprocals of each other. Tell students that you can find a reciprocal by “flipping” the order of the values in a ratio.
 - Now read through the description of part-to-whole ratios together. Make sure students recognize that, in this situation, the difference between the part-to-part and part-to-whole ratios is the second quantity.
 - Again use the model to show that one ratio describes 3 shaded parts to the whole, which is 5 total parts.
 - Challenge students to describe the model using a different part-to-whole ratio. Lead them to recognize that they can write 2 unshaded parts to 5 total parts, which is 2 to 5, $2 : 5$, or $\frac{2}{5}$.
 - Discuss that while a ratio can be written using a separating bar, it is only a true fraction if it names a part to a whole.

Lesson 1 Understand Ratios

Introduction

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Break Down the Skills

A **rate** compares two quantities with different units of measurement.

- You can write a rate in words or as a fraction.
- Here are some examples of rates.

\$10 for 5 notebooks or $\frac{\$10}{5 \text{ notebooks}}$	3 cups of oats for 4 servings or $\frac{3 \text{ cups of oats}}{4 \text{ servings}}$	60 miles in 2 hours or $\frac{60 \text{ miles}}{2 \text{ hours}}$
--	--	---

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Lesson 1 Understand Ratios

A unit rate is a rate in which the second measurement is 1 unit.

- You can write a rate as a unit rate by dividing.
 $\frac{60 \text{ miles}}{2 \text{ hours}} = 2 \frac{30 \text{ miles}}{1 \text{ hour}}$ ← 30 miles in 1 hour is a unit rate.

A rate is a special kind of **ratio**, which is a comparison of two quantities. There are two types of ratios. Use this model to compare them.

A part-to-part ratio compares one part of a whole to another part of the same whole.

3 shaded parts 2 unshaded parts

You can write the ratio of shaded parts to unshaded parts as 3 to 2 , $3 : 2$, or $\frac{3}{2}$.

A part-to-whole ratio compares one part of a whole to the total number of parts in the whole.

3 shaded parts 5 total parts

You can write the ratio of shaded parts to total parts as 3 to 5 , $3 : 5$, or $\frac{3}{5}$.

Equivalent ratios are ratios that can be simplified to the same value, such as in the same amount of these whole pies.

- The ratios $1 : 2$, $2 : 4$, and $4 : 8$ are equivalent.

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- Read aloud the definition of **equivalent ratios** and then invite volunteers to describe them in their own words.
- Point out that the model shows the same ratio of uneaten pie to the total pie in all three images. Explain that the difference is the number of parts in each image. Show students that the three ratios, therefore, have the same value.
- Read together the information about **ordered pairs**.
 - Remind students that they have learned about ordered pairs before when plotting points. Review that the first value is the **x-coordinate** and the second value is the **y-coordinate**.
 - Work together to follow the process for writing the ratios as ordered pairs.
- Ask students to each put a finger on the **origin** on the coordinate plane.
 - Discuss how the x -coordinate indicates the distance from the origin along the x -axis and the y -coordinate indicates the distance from the origin along the y -axis.
 - Guide students to plot the ordered pair $(1, 2)$ by moving 1 unit to the right of the origin and then 2 units above that point. Then repeat for the other ordered pairs.

Chapter 1 • Ratios and Rates

You can organize equivalent ratios in a table.

- Then you can write them as **ordered pairs**, which are sets of numbers written in a particular order.
- Remember that the first value in an ordered pair is the **x-coordinate**. The second value is the **y-coordinate**.

(x, y)

Ratio	First Value x-coordinate	Second Value y-coordinate	Ordered Pair
1 : 2 →	1	2	→ (1, 2)
2 : 4 →	2	4	→ (2, 4)
3 : 6 →	3	6	→ (3, 6)
4 : 8 →	4	8	→ (4, 8)

Look for patterns to fill in missing values or add to the table.

You can plot the ordered pairs on the coordinate plane.

- The x -coordinate tells how far the point is to the left or right of the **origin** $(0, 0)$.
- The y -coordinate tells how far the point is above or below the origin.

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GUIDED INSTRUCTION

Guide students through each activity. Read and discuss all the tips in conjunction with the related activities.

- Open the discussion with a review of area.
 - Confirm that students understand that area is the measure of the inside region of a plane figure and that area is measured in square units.
 - Clarify with students that area can be measured by using congruent unit squares. Explain that the units can be any unit of length, but they must all be the same.
 - Together, work through the steps for finding the area using a rectangular grid made of unit squares.
 - Discuss that area can be found by counting the unit squares or by multiplying the number of rows by the number of columns.
 - Remind students that a row goes across horizontally. Tell students to count the number of unit squares in each row and enter it in the blank. Then have them repeat for each column, making sure they understand that columns go up and down vertically.
 - Have students complete the equation that is used to find the area of the rectangle.
 - Ask students to count the number of unit squares to confirm the area they found using multiplication.
- Point out that when finding the area of rectangles, it is possible for the lengths and widths to be fractional values. Explain that for this reason, it is important to review what students know about fractions.

Lesson 1 • Understand Ratios

Guided Instruction

Area is a measure of the amount of space taken up by a plane figure.

One way to measure area is by using unit squares. A unit square is a square that has a side length of 1 unit.

The unit of a unit square can be any unit of length, such as inches or centimeters.

- One way to find the area of this rectangle is by counting the number of unit squares inside it.
- Another way to find area is to count the number of unit squares in each row and column. Then multiply the number of rows by the number of columns.

There are each row has 3 unit squares and each column has 4 unit squares.

The area of this rectangle is $\frac{3}{1} \times \frac{4}{1} = 12$ square units, which is the same answer you get by counting.

Count the unit squares. 12 unit squares.

Some areas are fractions, instead of whole numbers.

When working with fractions, it often helps to write equivalent fractions. Equivalent fractions are fractions that have the same value even though they have different numbers in them.

Use the model to fill in the equivalent fractions.

$\frac{1}{2} = \frac{2}{4} = \frac{3}{6}$

4 Level F Copying is prohibited.

- Remind students that a fraction compares a number of parts to the total number of parts in the same whole.
- Review that the top, or numerator, of a fraction shows the number of parts and the bottom, or denominator, shows the total number of parts in the whole.
- Say aloud the word *equivalent* and explain that it means “the same.” Point out that equivalent fractions have the same value.
- Direct students to the model that shows equivalent fractions. Make sure they realize that the numerator in each fraction describes the number of shaded parts, and the denominator describes the total number of parts for each fraction circle.
- Ask students to fill in the missing values to show the equivalent fractions. Review their answers as a group.

Common Errors

Some students may find the model of equivalent fractions confusing or think that it shows different amounts because each circle is divided into a different number of parts. To help alleviate this confusion, draw a circle where all students can see. Divide the circle into fourths, shade $\frac{1}{4}$, and discuss. Then draw additional lines to divide the same circle into eighths. Discuss that $\frac{2}{8}$ are now shaded, but that the size of the shaded portion has not changed.

- Explain that you can find an equivalent fraction by multiplying the numerator and denominator by the same number.
- Begin by directing students to look at the example on the left. Tell students to multiply the numerator, 3, by 5, and fill in the product they find as the numerator of the equivalent fraction.
- Then tell them to multiply the denominator, 5, by 5, and fill in the product they find as the denominator of the equivalent fraction.
- Tell students that three-fifths is equivalent to fifteen twenty-fifths and have students fill in the value in the sentence below the problem.
- Then tell students that they can also find equivalent fractions by dividing. Ask them to divide 12 and 16 by 4 to find an equivalent fraction. Have them fill in the missing values.
- Discuss how students can use what they know about fractions to learn about rates. As a group, read the definition of rate.
 - Invite a volunteer to read aloud the description of Elijah’s reading rate and have students fill in the numbers.
 - Then have students fill in the rate for the cost of gasoline. Confirm their responses.
- Point out that a unit rate is a special kind of rate in which a measurement is compared to a measurement of 1 unit.
 - As needed, clarify what this means with examples such as 1 mile, 1 hour, or 1 gallon.
 - Discuss how the steps used to determine a unit rate are the same as for determining an equivalent fraction.

Chapter 1 • Ratios and Rates

When you do not have images to look at, you can multiply or divide the numerator and denominator by the same number to find an equivalent fraction.

Fill in the missing numbers to find each equivalent fraction.

Equivalent fractions: $\frac{3}{5}$ and $\frac{15}{25}$ and $\frac{6}{8}$ and $\frac{3}{4}$

You can use what you know about fractions to describe rates. A rate compares two quantities that are measured in different units.

Write a rate for each example.

Elijah read 40 words in 2 minutes.

Gasoline costs \$16 for 6 gallons.

A unit rate is a rate in which the denominator is 1 unit. To calculate a unit rate, divide one quantity by the other.

The Li family drove 124 miles in 4 hours. Fill in the missing values to find the unit rate.

124 miles = $\frac{124}{4}$ miles = $\frac{31}{1}$ miles per hour

Complete the table to fill in each unit rate.

Rate	Unit Rate
48 beads, 6 bracelets	$\frac{8}{1}$ beads per bracelet
60 kilometers, 5 hours	$\frac{12}{1}$ kilometers per hour

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- Read aloud the unit rate problem about the Li family.
- Have students fill in the missing values. If students struggle, remind them that they must divide the numerator and denominator by the same number.
- Mention that the answer would still be correct without writing the 1. Remind students that if they see a unit of measure without a number, they know that the number is 1.
- Have students complete the table to write unit rates from the given rates. Review as a group.

Common Errors

Some students may be confused by how unit rates differ from rates in general. Work with them to brainstorm unit rates they might encounter in their daily lives, such as prices at the store or speed limit signs on the road. Challenge them to find examples then have students describe examples of unit rates in their own words.

- Discuss with students the difference between a rate and a ratio.
 - Have students give examples of rates and then ratios.
 - Explain that a ratio can be written in the forms: a to b , $a : b$, and $\frac{a}{b}$.
- Explain that some ratios, known as part-to-part ratios, compare one part of a whole to another.
 - Direct students to look at the model of the diamond shapes. As a group, determine that there are 4 shaded parts, 2 unshaded parts, and 6 total parts.
 - Ask students to fill in the ratios and then say them aloud.
- Explain that other ratios compare parts to the whole or the whole to parts.
 - Ask students to fill in the ratios and then say them aloud.
- Discuss that writing equivalent ratios is the same as writing equivalent fractions.
 - As a group, work through the examples of apples and oranges. Have students fill in the values. When finished, explain that the models show equivalent ratios. Explain that the first model shows 2 apples for 3 oranges, which is a ratio of 2 to 3. Then explain that the second ratio shows twice as many apples and oranges, which is a ratio of 4 to 6.
 - Point out that you can find equivalent ratios using multiplication and division. Remind students that they must multiply or divide both numbers in a ratio by the same amount. Direct them to fill in the missing values to complete the equivalent ratio statements.

Lesson 1 • Understand Ratios

A rate is a kind of ratio. A ratio compares two quantities.

- A rate compares quantities with different units.
- A ratio compares quantities with the same or different units.

Use the model to describe the ratios.

Write part-to-part ratios.


- Shaded parts to unshaded parts: $\frac{4}{2}$ to $\frac{2}{4}$
- Unshaded parts to shaded parts: $\frac{2}{4}$ to $\frac{4}{2}$


Write part-to-whole and whole-to-part ratios.

- Shaded parts to all parts: $\frac{4}{6}$
- Unshaded parts to all parts: $\frac{2}{6}$ to $\frac{6}{6}$
- All parts to shaded parts: $\frac{6}{4}$ to $\frac{6}{2}$
- All parts to unshaded parts: $\frac{6}{2}$

Just as you did with equivalent fractions, you can describe equivalent ratios.

Fill in the equivalent ratios of apples to oranges.

2 : 3 

4 : 6 

You can use multiplication or division to write equivalent ratios.

Fill in the missing numbers to create equivalent ratios.

$\frac{2}{3} = \frac{4}{6}$ $\frac{3}{4} = \frac{6}{8}$

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Common Errors

When students see a ratio such as 2 : 3, some of them may attempt to find an equivalent ratio by adding the same number to the numerator and denominator. For example, they may think 2 : 3 is equivalent to 4 : 5 because they added 2 to both the numerator and denominator. Remind students that the Identity Property of Multiplication states that the *product* of a factor multiplied by 1 is equivalent to the factor. Write a ratio on the board where all students can see, such as $\frac{2}{3}$. Draw arrows from the numerator and denominator of the ratio with a multiplication sign near the arrows. Have a volunteer write the same number next to each multiplication sign and then write the equivalent ratio. Repeat with more ratios until students feel comfortable with this concept.

- Introduce the idea of organizing ratios in a table.
 - Have a volunteer read aloud the problem about the ratios of boys to girls in a class.
 - Discuss what a ratio of 3 to 5 means. Ensure that students understand that for every 3 boys, there are 5 girls.
 - Instruct students to complete the ratio table by multiplying both terms by each successive whole number.
 - Invite students to share their answers with the class.
- Point out to students that they created tables when they used ordered pairs. Review that an ordered pair is made up of two values, and that the first value represents the x -coordinate, while the second value represents the y -coordinate.
 - Explain that the two values in a ratio can be written as the two coordinates in an ordered pair.
 - Work as a group to complete the table to show the ratios and ordered pairs below 3 to 2.
- Remind students that they can plot ordered pairs on a coordinate plane.
 - Review that each coordinate represents a distance from the origin, which is the point $(0, 0)$.
 - Guide students to plot the first ordered pair on the coordinate plane. Direct students to begin at the origin and move right for the x -coordinate and up for the y -coordinate.
 - Have students work in pairs to plot the remaining ordered pairs.
 - Ask students to describe the points plotted from the ordered pairs. Confirm that the ordered pairs formed from equivalent ratios make up a line.

Chapter 1 • Ratios and Rates

You can use tables to organize and find equivalent ratios.

The ratio of boys to girls in a class is 3 : 5. Complete the ratio table to compare boys to girls.

	Boys	Girls	
	3	5	
$3 \times 2 \rightarrow$	6	10	$\leftarrow 5 \times 2$
$3 \times 3 \rightarrow$	9	15	$\leftarrow 5 \times 3$
$3 \times 4 \rightarrow$	12	20	$\leftarrow 5 \times 4$

You can use the values in a ratio table to write ordered pairs.

Complete the equivalent ratio table for the ratio 3 to 2. Then write the ordered pair.

First Value	Second Value	Ordered Pair
3	2	$(3, 2)$
6	4	$(6, 4)$
9	6	$(9, 6)$
12	8	$(12, 8)$

Now that you have ordered pairs, you can plot them on a coordinate plane.

Plot the ordered pairs from the equivalent ratio table on the coordinate plane.

Remember that in an ordered pair, the first value is the x -coordinate, and the second value is the y -coordinate.

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Common Errors

Students may confuse the order of the coordinates when plotting ordered pairs. Explain that the first coordinate represents the horizontal or x -coordinate, and the second coordinate represents the vertical or y -coordinate. Point out that the coordinates are in ABC order within an ordered pair (x comes before y). Have students write an ordered pair as (x, y) to help them remember. You may also wish to have students use Copy Master 2 at the end of these teacher notes for additional practice. Make a copy of the master, insert a ratio or a completed ratio table for each problem, and make your student copies. Have students write the values as ordered pairs and plot them on the coordinate grids. By providing different starting ratios or tables, the activity can be completed over and over and be a different activity each time.

EXIT TICKET

- Have students fill in the Exit Ticket. Read through the word problem together. Make sure students understand that they have been provided a rate that can be used to write equivalent ratios. Clarify that the equivalent ratios can then be translated into ordered pairs that they are to plot on the coordinate plane.

Lesson 1 • Understanding Ratios

Exit Ticket

A snowstorm lasted for 5 hours. When the storm ended, 10 inches of snow had fallen. The same amount of snow fell each hour.

What is the unit rate of snowfall in inches per hour?

2 inches per hour

Complete the ratio table to show the number of inches of snow during each of the first 4 hours.

Time (hours)	Snowfall (inches)
1	2
2	4
3	6
4	8
5	10

Create ordered pairs from the ratio table and plot them on the coordinate plane.

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ADDITIONAL SUPPORT

SUPPORT FOR STRUGGLING LEARNERS

- For students who struggle with accurate multiplication or accurate division, provide a multiplication table to check their work when finding equivalent ratios or unit rates.
- Give students counters or tiles to practice writing ratios. Have students draw the counters or tiles, and then write part-to-part, part-to-whole, and whole-to-part ratios. Students can then share their drawings with a partner and challenge the partner to write the ratios. The students can then compare their answers and correct any differences.
- Some students are likely to struggle with making and organizing sets of equivalent ratios. Give such students extra practice writing equivalent ratios by completing Copy Master 1 at the end of these teacher notes. Make a copy of the master, insert a ratio at the top of each of the two tables, and make your student copies. By providing different starting ratios, the activity can be completed over and over and be a different activity each time.

SUPPORT FOR ENGLISH LEARNERS

- English learners might be confused about the difference between *unit squares* and *square units*. Point out that while the names are similar, they are not the same. Explain that unit squares are used to tile a rectangle and identify the area and square units are the units of measurement for area. Have students draw a diagram and label the area in square units and shade a unit square.
- English learners may struggle with the vocabulary for plotting points in a coordinate system. Have students create flashcards for the terms *coordinate plane*, *origin*, *ordered pair*, *x-axis*, *y-axis*, *x-coordinate*, and *y-coordinate*.
- Spanish cognates: equivalent/equivalente, fraction/fracción, area/área, coordinate/coordinar, unit/unidad

EXTENSION ACTIVITIES

- Challenge students to write part-to-part and part-to-whole ratios using different sets, such as consonants and vowels, or classroom objects, such as books.
- Have partners generate their own word problems that include unit rates. Suggest that they write problems about buying clothes, saving money, hiking, or other situations that form rates. Point out that they could also increase the number of servings in a recipe. Encourage pairs to exchange their problems with another pair and solve.
- Invite students to discuss why equivalent fractions cannot be formed by adding or subtracting the same number to the numerator and denominator.

SAMPLE

Name _____

Date _____

Find equivalent ratios.

Ratio: _____

First Value	Second Value

Ratio: _____

First Value	Second Value

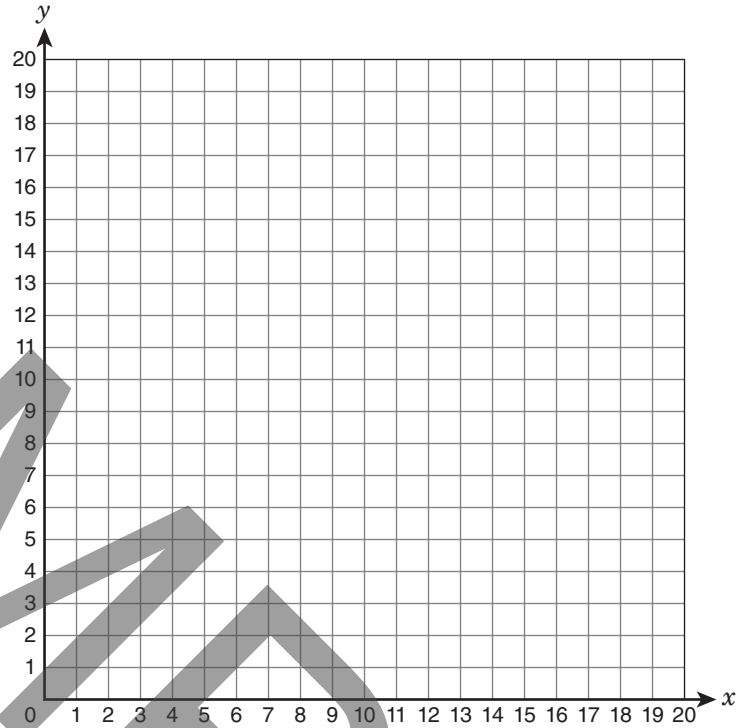
Name _____

Date _____

Plot equivalent ratios on the coordinate plane.

Ratio: _____

First Value	Second Value



Ratio: _____

First Value	Second Value

